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Determining the Surface Water Exchange between the Kafue River and Lukanga Swamps in the Central Province of Zambia

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Abstract The Lukanga swamps are a part of the Kafue River catchment. It lies about 30km to the east of the Kafue River. It is connected to the Kafue River mainly through the Munwinu and Lukanga channels and during peak floods some waters of the Kafue River are said to back into the Lukanga swamps. This study thus modelled the surface water exchange between the Kafue River and the Lukanga swamps in order to understand the surface flow interactions between the Kafue River and the Lukanga swamps. The modelling employed graph theory through which the water system was recognised as a graph of 3 nodes and 3 edges. Historical water levels observed over a period of 81 months were used as input variables whereas elevations obtained from channel profiles from a corrected SRTM DEM were used as the input constants. The elevations represented channel floor. The results showed that there were 56 months in which the network had flow in all the edges, that the Lukanga channel flowed throughout the year from the Lukanga swamps to the Kafue River and that the Munwinu channel only flowed from the Kafue River to the Lukanga swamps whenever there was flow. Thus the Munwinu channel as well as the Lukanga channel does not present bidirectional flow at all.

Keywords: *graph theory, Lukanga swamps, Kafue River, surface water exchange, surface water modelling*

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1. Introduction

Wetlands are biologically diverse and productive ecosystems which support a variety of plant life, diverse communities of invertebrates, vertebrates and carnivores. They thus maintain different communities of ecological and economic value [1] influenced by the primary factor which is the movement, distribution and quality of water [2]. Consequently the balance of water inflows and outflows together with geomorphology and soils determine when, for how long and how the wetlands flood, to in turn determine their productivity [3]. Wetlands therefore have a significant influence on the hydrological cycle [4]. The Lukanga swamps Kafue River area is one such wetland where water movement, distribution and quality matters.

The Lukanga swamps and the Kafue River experience some surface water exchange between them but it is not well understood, hence the need to model it since water models tell a lot about the various issues concerning water resources management and engineering. In all these models topography is an important land surface characteristic that affects most aspects of the water balance including generation of surface and subsurface runoff, flow paths and the rate of water movement [5,6,7].

The Lukanga swamps Kafue River area was thus modelled as a spatial graph using elevation as a single independent variable since bathymetric and topographic information (elevation) is key to the development of reliable hydraulic models [8]. [9] predicted mean annual stream flows using only mean basin elevation in a simple linear regression. This information often comes in the form of accurate land-surface elevation data.

This study modelled the movement and direction of movement of surface runoff along the Kafue River, the Munwinu channel, the Lukanga swamps and the Lukanga channel in form of a graph in which water flows were translated into mathematical expressions that aided in analysing the surface water exchange between the Kafue River and the Lukanga swamps.

Thus only the flow and direction of that flow of the surface water to and from the Kafue River from the Lukanga swamps was looked at with respect to the variation of water levels. [10,11,12,13] all observed or heard that the Kafue River pushed some water into Lukanga swamps at high flows. The developed model therefore attempted to answer this assertion. An endeavour was also made to relate water level data to actual elevation by analysing historical gauging stations establishment information where available.

The surface water exchange model specifically models the flow and its direction using water levels as input and presence and direction of flow as output.

2. Data and Methods

The data used in this study comprised water level data from selected river gauge stations and channel bed elevations from points of interest along the channels of interest whose profiles were knocked off an SRTM digital elevation model. The digital elevation model was corrected with GNSS levels measured across the study area (Figure 4). Modelling of the water exchange was done using concepts of graph theory.

2.1. Elevation Data

Topology is the governing factor determining the flow of water over a network [14,15]; hence height differences in the network are of paramount importance in any water model. That is the reason why elevations were at the centre of this model. Elevation data was obtained from SRTM Digital Elevation Model that was corrected with GNSS field data. The GNSS data was measured in static mode and post processed to obtain ellipsoidal heights which were then corrected using ZG2016 a gravity model improved by [16] from ZG96 that was developed by [17].

Stream paths were generated automatically but with some manual intervention over the digital elevation model. Thereafter the profile paths of Munwinu and Lukanga channels were generated as shown in Figure 1 and Figure 2. Elevations of interest (Table 1) were then read out from the profiles. These being elevations for the Kafue River at Munwinu channel confluence, Kafue River at Lukanga channel confluence and the Lukanga swamp.

Since the 2 profiles showed that there were elevations higher than both the Lukanga swamps and the Kafue River, elevations for the highest points along the profiles were also recorded. The points of interest for which elevation data was obtained coincided with what later were considered as nodes of the network of the water system.

The obtained elevations were assumed to represent their respective river beds. These data were used in conjunction with water level data from selected gauging stations in the study area.

2.2. Water Level Data

The Department of Water Affairs (DWA) and now the Water Resources Management Authority (WARMA) of Zambia had installed gauging stations in the study area. Figure 3, an extract from the Hydrological Survey of Zambia map, shows gauging stations that existed in the study area as at 1968. The stations in the vicinity of the study area are listed in Table 2. Most of these stations are no longer operational but their historical data was archived. Only one of the listed stations is still operational though. Water level data were obtained from WARMA with their kind permission.

Although 6 gauging stations were found to be within the area of interest (Table 2), only 3 were used, namely, stations 4350 (Chilenga) upstream of the Lukanga swamps on the Kafue river, 4425 (Munkunkwa) at the exit of the Lukanga channel from the Lukanga swamp and 4431 (Mongo) at the confluence of the Lukanga channel and the Kafue river, to determine the exchange of flow between the Kafue River and the Lukanga swamps. It was assumed that water levels at station 4350 represented water levels at Kafue-Munwinu confluence, water levels at station 4425 represented water levels at Lukanga swamps while water levels at station 4431 represented water levels at Kafue-Lukanga confluence. These water level data were used with base elevations of the relevant junctions which were assumed to represent river bed elevations.

The water level data for these stations have many gaps in the daily water levels recorded at these stations for one reason or the other such that it was difficult to find common dates data across the stations of interest that covered longer periods.

The water exchange model uses the observed water levels as its variables since they were read every day and varied from time to time. The water levels used in this study had to be from common dates for all the 3 gauging stations, i.e. data from dates where there were records for all the three stations used. These data were found to be from the following dates only:

- 01.10.1962 – 29.09.1968
- 02.01.1969 – 29.09.1969

These data, which represented 81 months (6.75years) of data, were reduced to monthly means for easy handling as variables for input into the water exchange model (Table 3). This way the resulting graphs represented monthly scenarios.

2.2.1. Gauge Stations Locational Data

These gauging stations were established using arbitrary elevations (with local benchmarks) which had no relationship with the national datum at all as their use was only for reading off water levels at a particular location to facilitate calculation of discharge flows. Even their locational data was scaled off topographic maps (Figure 3) for rough indication of where they were located [18]. It therefore followed that water levels at different locations did not spatially relate to each other at all.

An attempt to allocate them mean sea level elevations in this study was not successful for it was difficult to determine the zero point of the gauge plates since the bench marks and in most cases the gauge stations themselves are no longer in existence and to re-establish their positions was not possible given the crude locational information they have or don't have at all (Table 2).

2.3. Graph Theory

The model was developed using concepts of graph theory [19,20,21,22]. Graph theory is a branch of discrete mathematics and systems theory widely used in many scientific disciplines [23,24,25,26] to represent physical networks such as electrical circuits or less tangible interactions such as in databases. Any object's mathematical representation by points and connections may be called a graph.

A graph is therefore a data structure consisting of a set of nodes connected by edges. In spatial systems nodes may be locations or objects in space or indeed their properties. Edges may represent spatial relationships or processes occurring between locations or objects in space [27]. As a result a graph represents a network system and that network could be a river network. An extension of river networks is a surface graph which identifies important points such as pits, peaks and links such as slope lines and curvature isolines. Surface graphs enable cartographic representation of relief and its generalisation [28,29,30].

A graph is typically represented by an adjacency matrix which is what is normally made use of in graph theory. A graph could also be spatially explicit when its layout is determined by spatial referencing of nodes [31,32] where spatial objects are linked by water fluxes. Edges may have weights which could represent proportion of flow, capacity of flow or resistance of flow [33,34,35]. Spatial objects in a network could be aggregated using network aggregation such as node and edge contraction in order to identify clusters in larger graph structures based on their attributes [36,37].

The dynamics of surface water exchange in this study were thus represented as a graph in order to analyse the interactions of the water flow between the Lukanga swamps and the Kafue River using elevations and water levels as constants and variables respectively.

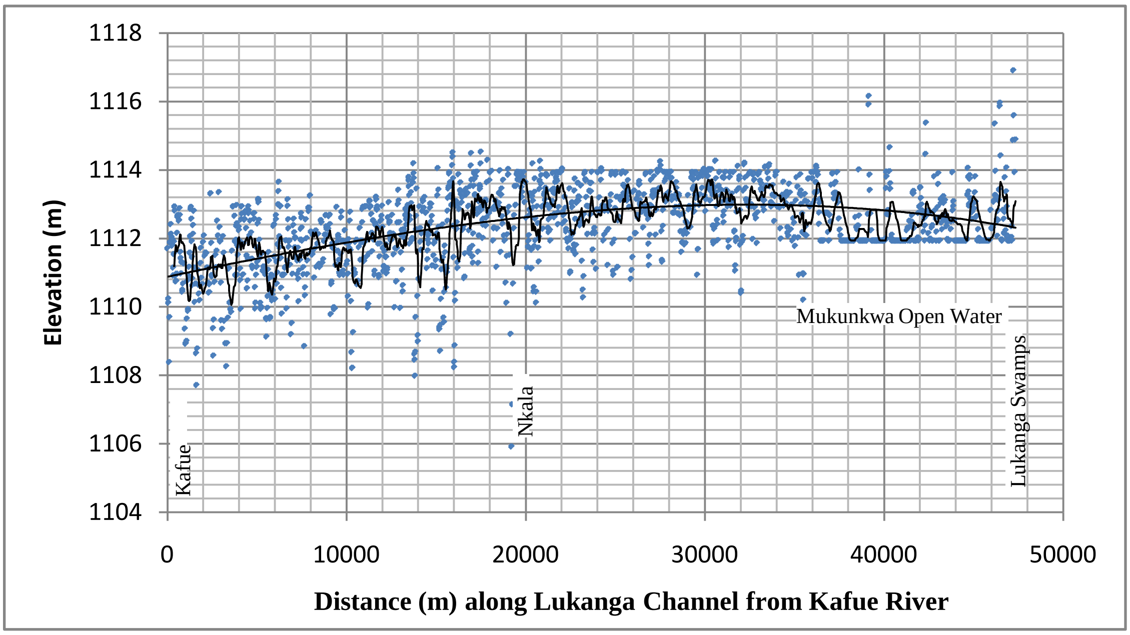

Figure 1. Lukanga channel profile (Kafue River to Lukanga swamps)

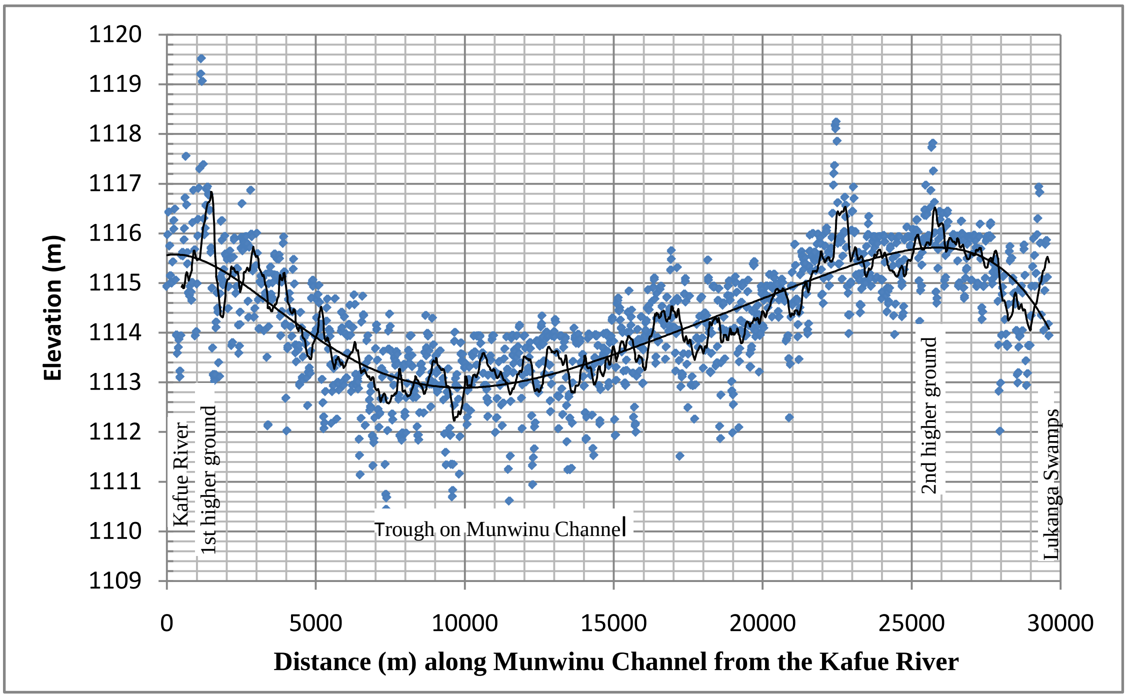

Figure 2. Munwinu channel profile (Kafue river to Lukanga Swamps)

Table 1. Base elevations (constants) used in the model

	Constant	Elevation (m)	vertex	Location
1	Hv_1	1114.90	v_1	Kafue-Munwinu junction
2	Hv_2	1112.95	v_2	Lukanga swamps junction
3	Hv_3	1110.20	v_3	Kafue-Lukanga channel junction
4	X_l	1113.70	intermediate	Highest elevation along edge
5	X_m	1116.90	intermediate	Highest elevation along edge

Table 2. Gauging stations within the study area (courtesy of WARMA)

Ref. No.	Station Description	Latitude	Longitude	Bench Mark	Operating
4350	Kafue @ Chilenga	-14.10000	27.41667	5.515 & 7.495	Yes
4390	Lukanga swamp @ Chilwa Island	-14.21667	27.65000	Unknown	No
4400	Lukanga swamp @ Kapukupuku	-14.61678	27.94997	Unknown	No
4425	Lukanga channel @ Mukunkwa	-14.41667	27.50000	Unknown	No
4430	Lukanga channel @ Mongo	-14.36667	27.00000	Unknown	No
4431	Kafue @ Lukanga channel confluence	-14.36667	27.16667	Unknown	No

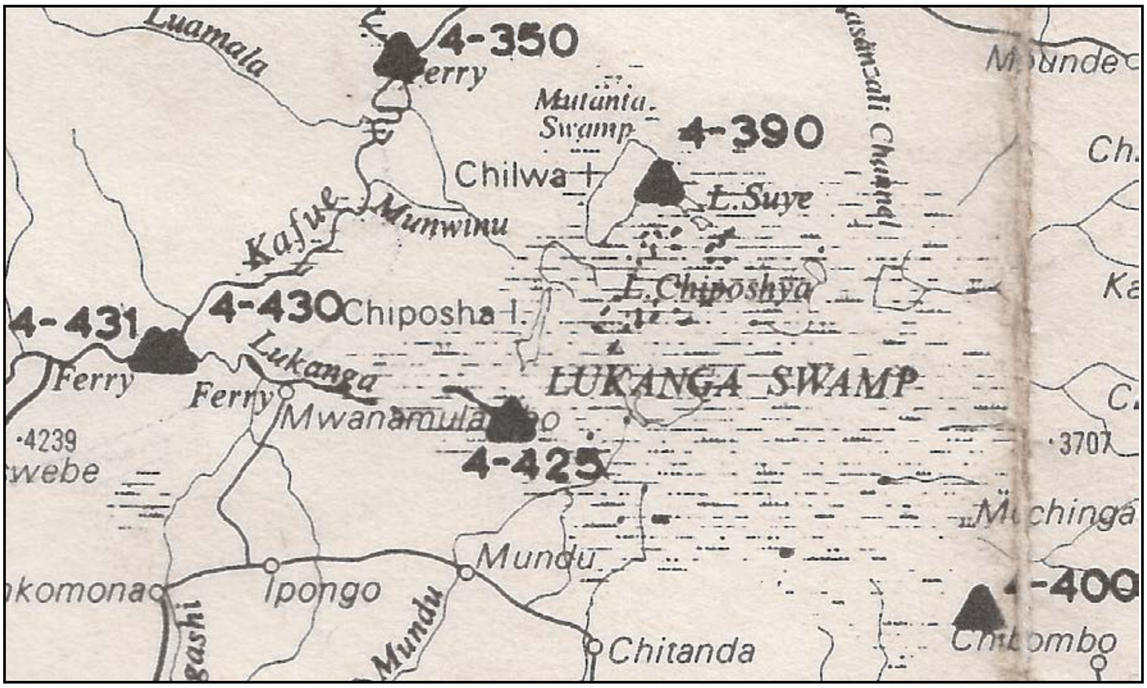

Figure 3. Hydrological Survey of Zambia extract for gauge stations in study area [38]

3. Surface Water Exchange Model

The area between the Lukanga swamps and the Kafue River was first recognised as a graph and its configuration reduced as a graph [39] (Figure 4) after identifying its nodes and edges. Thereafter the vectors were determined and the equations representing water flow developed.

The Lukanga swamps and the Kafue River were joined together mainly by two channels, namely the Munwinu channel upstream of the Kafue River and the Lukanga channel downstream of the Kafue River, which together formed a network along which water flowed to the swamp from the river, and vice-versa. The nodes were taken as the points at which the two channels joined the Lukanga swamps and the Kafue River.

Hence the identified network (Figure 4) was represented as a graph with four nodes (Figure 5) as follows:

a) Node 1: Kafue River - Munwinu channel junction
b) Node 2: Munwinu channel (Mukumbang'ombe) – Lukanga swamps junction
c) Node 3: Lukanga swamps – Lukanga channel (Mukunkwa) junction
d) Node 4: Lukanga channel – Kafue River junction

Figure 4. Recognising the identified network as a graph (image courtesy of Google Earth)

The edges were the links between the identified nodes.

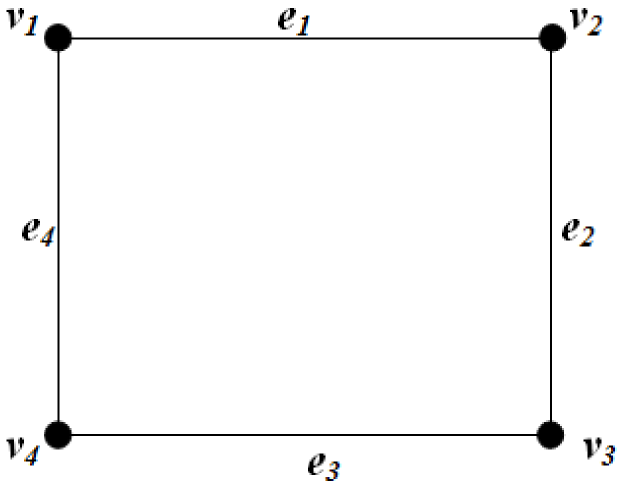

Figure 5. Abstracted graph from the network in Figure 2

In addition to the 4 nodes, 1 point was identified on each of the 2 channels connecting Lukanga swamps to the Kafue River. These points were taken at a location which had the highest elevation along the profile of a particular channel but were not designated as nodes but as constraints on their respective edges. Figure 5 shows the abstracted graph from the identified nodes and edges in the network of Figure 4. But because the second and third nodes represented the same object, the Lukanga swamps, they were contracted into one node to yield a new graph (Figure 6) with only 3 nodes and 3 edges [36,37]. This node contraction was based on the assumption that the simplified water level within the swamp, as a singular body, was expected to be the same throughout the swamp.

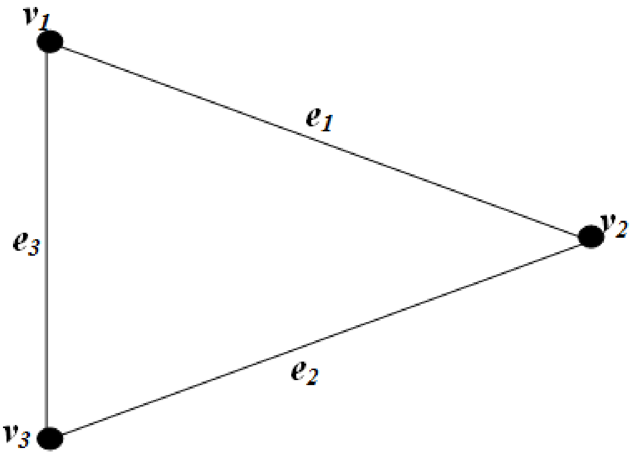

Figure 6. Contraction of Figure 3 into a 3 nodes - 3 edges graph

where, v_1, v_2, v_3 are vertices representing Munwinu confluence with Kafue River, Lukanga swamps and Lukanga channel confluence with Kafue River respectively and e_1, e_2, e_3 are edges.

The abstracted graph of Figure 6 is undirected but preliminary investigations showed it was possible to have the following scenarios [11,12,13] where:

- Water moved from the Kafue River to the Lukanga swamps.
- Water moved from the Lukanga swamps to the Kafue River.
- No water moved from or to either the Lukanga swamps or the Kafue River.

Implementing the above scenarios resulted in directed or mixed graphs [40]. However, from the preliminary investigations cited above there was a possibility to have the directed graph of three nodes and five directed edges

(Figure 7) resulting from the undirected graph of Figure 6. Derivation of relevant model equations was actually based on the directed graph of Figure 7 but implemented using the underlying graph which is Figure 6.

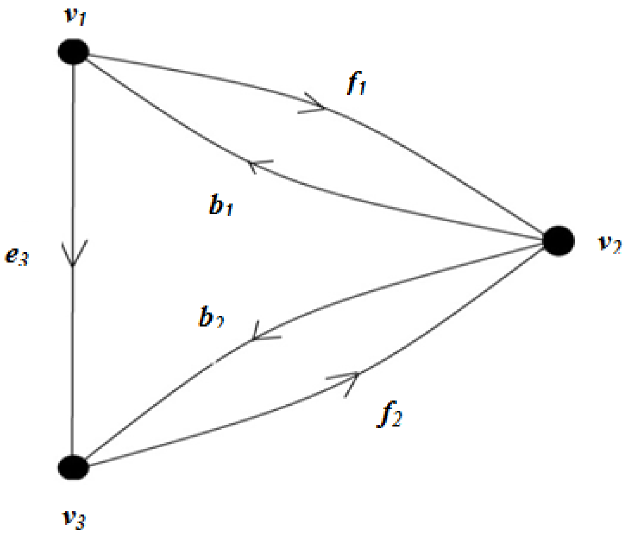

Figure 7. Directed graph showing 5 possible flow directions

where, ***f₁*** and ***b₁*** are forward and reverse vectors respectively of edge ***e₁*** and, ***f₂*** and ***b₂*** are forward and back vectors respectively of edge ***e₂*** and, ***e₃*** has only a forward vector because Kafue river always flows in one direction -downstream

3.1. Model Equations Derivation

When a profile is taken along a walk ***v₁, e₁, v₂*** of the graph in Figure 6, elevations along this walk could be extracted from the profile since topography is the determining factor in this study. This profile represented by Figure 8 was used to derive important parameters used in formulating equations for the determination of the presence and direction of flow of water along the Kafue River, Munwinu and Lukanga channels. The mean sea level (msl) elevations constituted constants whereas the observed water levels constituted the variables for the derived equations. The walk ***v₁, e₁, v₂*** represented the profile along the Lukanga channel. Figure 8 is a simplified version of the profiles in Figure 1 and Figure2.

Using this generalized profile of Munwinu channel the following constants and variables were obtained for the underlying graph in Figure 6:

Variables, ***Wv₁*** – water level at vertex ***v₁***
Wv2 – water level at vertex v2
Wv3 - water level at vertex v3
Constants, ***Hv₁*** – river bed elevation at vertex v1
Hv2 – swamp bed elevation at vertex v2
Hv3 – river bed elevation at vertex v3
Xm – highest point along Munwinu channel
Xl – highest point along Lukanga channel

Therefore:

The flow from Kafue River into Lukanga swamp along the Munwinu channel (***f₁***) only happens when the following conditions are satisfied:

$$\begin{aligned}&(Hv_1 + Wv_1) - X_m > 0\\ &\mathit{and}\ (Hv_1 + Wv_1) - (Hv_2 + Wv_2) > 0.\end{aligned} \tag{1}$$

The flow from Lukanga swamp into Kafue River along the Munwinu channel (b1) only happens when the following conditions are satisfied:

$$\begin{aligned}&(Hv_1 + Wv_1) - X_m < 0\\ &\mathit{and}\ (Hv_1 + Wv_1) - (Hv_2 + Wv_2) < 0.\end{aligned} \tag{2}$$

It therefore follows that the following conditions represent the forward and reverse water flow along the Lukanga channel:

$$\begin{aligned}&(Hv_3 + Wv_3) - X_l > 0\\ &\mathit{and}\ (Hv_3 + Wv_3) - (Hv_2 + Wv_2) > 0.\end{aligned} \tag{3}$$

$$\begin{aligned}&(Hv_3 + Wv_3) - X_l < 0\\ &\mathit{and}\ (Hv_3 + Wv_3) - (Hv_2 + Wv_2) < 0.\end{aligned} \tag{4}$$

Consequently, it means that water along edges ***e₁*** and ***e₂*** could flow in a forward or reverse direction as per conditions set in equations (1) to (4). But it is also possible that there could be no flow at all in either direction. This meant that there was either a flow in either of edges ***e₁*** and ***e₂*** or there was none. Thus using Boolean algebra '1' represented presence of flow and '0' absence of flow with an addition of "-1" to represent when flow is reversed, such that using forward and reverse vectors shown in Figure 8, equations (1) to (4) were reformulated as follows:

$$f_1 = \begin{cases}\mathbf{1\ if}\ (Hv_1 + Wv_1) > X_m > (Hv_2 + Wv_2),\\ \mathbf{0\ otherwise}\end{cases} \tag{5}$$

$$b_1 = \begin{cases}\mathbf{-1\ if}\ (Hv_1 + Wv_1) < X_m < (Hv_2 + Wv_2),\\ \mathbf{0\quad otherwise}\end{cases} \tag{6}$$

$$f_2 = \begin{cases}\mathbf{1\ if}(Hv_3 + Wv_3) > X_l > (Hv_2 + Wv_2),\\ \mathbf{0\ otherwise}\end{cases} \tag{7}$$

$$b_2 = \begin{cases}\mathbf{-1\ if}(Hv_3 + Wv_3 < X_l < (Hv_2 + Wv_2),\\ \mathbf{0\quad otherwise}.\end{cases} \tag{8}$$

Note that for edges ***e₁*** and ***e₂*** the reverse vectors have been assigned '-1' instead of just '1' in order to automatically determine the direction of flow. In addition ***e₃*** also only has 2 possibilities of either a forward flow or no flow at all, since water only flows downstream and is represented as follows:

$$f_2 = \begin{cases}\mathbf{1\ if}\ (Hv_1 + Wv_1) > (Hv_3 + Wv_3),\\ \mathbf{0\ otherwise}.\end{cases} \tag{9}$$

Thus the possible values for these variables are the Boolean numbers -1, 0, and 1 [41,42] which were possible returns of the conditions of the equations (5) to (9).

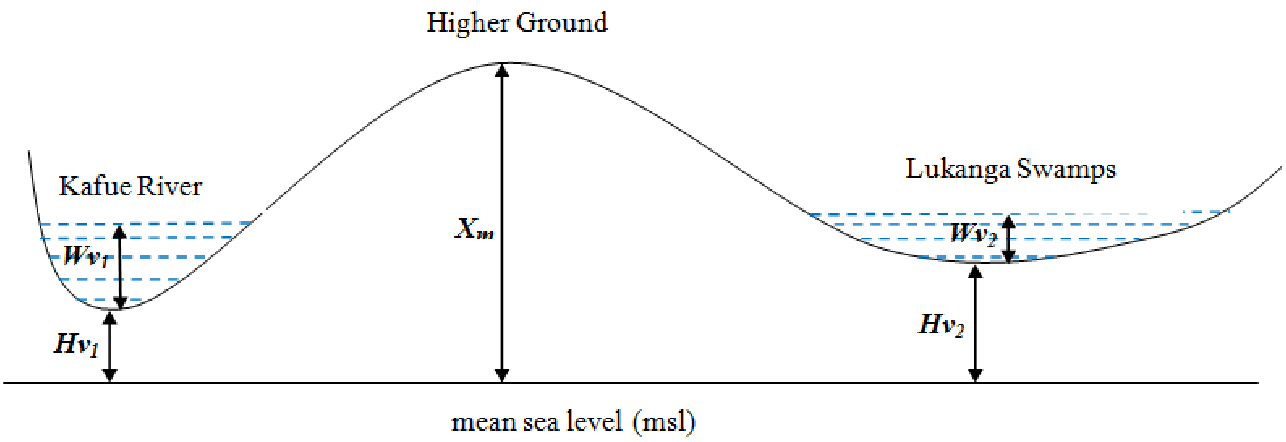

Figure 8. Generalised profile for Munwinu and Lukanga channels (from Kafue River to Lukanga Swamps)

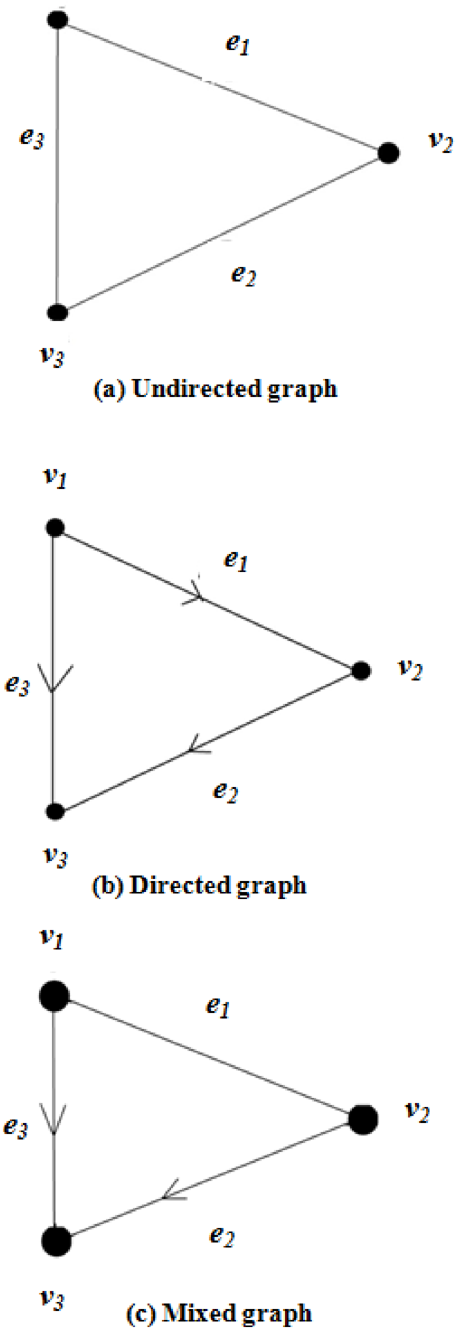

Figure 9. Possible graphs (a, b, c) resulting from equations (5) to (9)

The results of equations (5) to (9) were then applied to the underlying graph (Figure 6) to depict the direction of water flow on each edge. The results were presented as:

a) An undirected graph for cases where all the 3 edges had no flow in them.
b) A directed graph for cases where all the 3 edges had flow in any direction.
c) Mixed graph where some edges had no flow yet still others had flow in a any direction.

These graphs represented varying interactions of water flow between the Kafue River and the Lukanga swamps through the Munwinu and Lukanga channels, Lukanga swamps and Kafue river network.

3.2. Model Implementation

The water exchange model was developed to make use of observed water levels to typify what the water exchange would be between the Lukanga swamps and Kafue River through the Munwinu and Lukanga channels. The water levels were used with respect to mean sea level elevations. This meant that the observed water levels were to be converted to mean sea level elevations at vertices v_1, v_2 and v_3 in order to determine if there was any flow taking place in any of the edges e_1, e_2 and e_3. The inputs, therefore, into the model were the elevations at vertices v_1, v_2 and v_3 and the highest points (could be considered as intermediate vertices) X_l and X_m along edges e_1 and e_2 as constants for the model. Water levels were thus the variables of the model.

Vertices X_l and X_m, were not taken as vertices as such although they are intermediate vertices which did not participate in the model as such. They were used as constraints [43,44,45] in the model which imposed certain conditions that were necessary for the proper functioning of the developed model. The elevations at the vertices represented channel floor at their respective vertices such that adding an observed water level at such vertices resulted in the mean sea level elevation of the water level at that vertex.

In order to implement the model, equations (5) – (9) were reworked for use in MS Excel to calculate the result of each edge of the model graph. Consequently, the forward and reverse equations were combined into 1 logical test such that the final equations used were reduced to only 3 instead of 5, representing the 3 edges of the underlying graph (Figure 4), namely equations (10) to (12). These 3 logical tests were then fed with the 81 mean monthly water levels (Table 3) to generate presence or absence and direction of flow on each edge of the underlying graph's edges as '1', '-1' and '0', where '1' represented forward flow, '-1' reverse flow and '0' absence of any flow, defined as from Kafue River to Lukanga swamps in Figure 7.

Table 3. Mean monthly water levels and edge results

	4425	**4431**	**4350**			
Month & Year	**Mukunkwa**	***KL Confluence**	**Chilenga**	**Edge Results**		
	Level (m)	**Level (m)**	**Level (m)**	**Edges e_1, e_2 and e_3 Results**		
	Mean	Mean	Mean	e_1	e_2	e_3
Oct-62	2.302	2.116	1.847	0	-1	1
Nov-62	2.275	2.098	1.797	0	-1	1
Dec-62	2.329	2.467	4.047	1	-1	1
Jan-63	2.404	3.107	6.514	1	-1	1
Feb-63	2.544	3.387	6.890	1	-1	1
Mar-63	2.686	3.540	7.298	1	0	1
Apr-63	2.676	3.451	6.706	1	-1	1
May-63	2.588	3.115	5.441	1	-1	1
Jun-63	2.533	2.639	4.158	1	-1	1
Jul-63	2.503	2.448	3.550	1	-1	1
Aug-63	2.470	2.326	2.963	1	-1	1
Sep-63	2.433	2.202	2.284	1	-1	1
Oct-63	2.392	2.105	1.752	0	-1	1
Nov-63	2.381	2.160	2.098	1	-1	1
Dec-63	2.413	2.415	3.482	1	-1	1
Jan-64	2.438	2.609	4.479	1	-1	1
Feb-64	2.506	2.977	5.970	1	-1	1
Mar-64	2.530	3.182	6.286	1	-1	1
Apr-64	2.483	2.858	5.053	1	-1	1
May-64	2.433	2.366	3.284	1	-1	1
Jun-64	2.398	2.196	2.486	1	-1	1
Jul-64	2.368	2.141	2.174	1	-1	1
Aug-64	2.470	2.326	2.963	1	-1	1
Sep-64	2.304	2.032	1.538	0	-1	1
Oct-64	2.259	1.963	1.259	0	-1	1
Nov-64	2.231	1.971	1.303	0	-1	1
Dec-64	2.234	2.103	1.889	0	-1	1
Jan-65	2.300	2.522	4.351	1	-1	1
Feb-65	2.346	2.855	5.750	1	-1	1
Mar-65	2.325	2.846	5.709	1	-1	1
Apr-65	2.289	2.595	4.750	1	-1	1
May-65	2.255	2.205	3.032	1	-1	1
Jun-65	2.229	2.055	2.202	1	-1	1
Jul-65	2.107	2.092	1.724	0	-1	1
Aug-65	2.081	2.050	1.465	0	-1	1
Sep-65	2.156	1.953	1.383	0	-1	1
Oct-65	2.122	1.911	1.189	0	-1	1
Nov-65	2.092	1.923	1.205	0	-1	1
Dec-65	2.087	2.074	1.925	0	-1	1
Jan-66	2.093	2.192	2.549	1	-1	1
Feb-66	2.141	2.521	4.120	1	-1	1
Mar-66	2.194	2.866	5.459	1	-1	1
Apr-66	2.179	2.794	5.029	1	-1	1
May-66	2.157	2.362	3.114	1	-1	1
Jun-66	2.131	2.161	2.121	1	-1	1

	4425	**4431**	**4350**			
Month & Year	**Mukunkwa**	***KL Confluence**	**Chilenga**	**Edge Results**		
	Level (m)	**Level (m)**	**Level (m)**	**Edges e_1, e_2 and e_3 Results**		
	Mean	Mean	Mean	e_1	e_2	e_3
Jul-66	2.107	2.092	1.724	0	-1	1
Aug-66	2.081	2.050	1.465	0	-1	1
Sep-66	2.052	1.999	1.268	0	-1	1
Oct-66	2.015	1.943	1.056	0	-1	1
Nov-66	1.985	1.923	0.978	0	-1	1
Dec-66	1.970	1.981	1.188	0	-1	1
Jan-67	2.012	2.213	2.295	1	-1	1
Feb-67	2.040	2.449	3.561	1	-1	1
Mar-67	2.098	2.804	5.107	1	-1	1
Apr-67	2.099	2.984	5.849	1	-1	1
May-67	2.080	2.724	4.766	1	-1	1
Jun-67	2.053	2.236	2.664	1	-1	1
Jul-67	2.028	2.105	2.002	1	-1	1
Aug-67	2.002	2.050	1.666	0	-1	1
Sep-67	2.156	1.953	1.383	0	-1	1
Oct-67	1.935	1.972	1.188	0	-1	1
Nov-67	1.923	2.041	1.383	0	-1	1
Dec-67	1.936	2.227	2.295	1	-1	1
Jan-68	1.965	2.474	3.527	1	-1	1
Feb-68	1.993	2.863	5.268	1	-1	1
Mar-68	2.024	2.852	5.111	1	-1	1
Apr-68	2.005	2.752	4.627	1	-1	1
May-68	1.973	2.403	3.070	1	-1	1
Jun-68	1.943	2.185	2.114	1	-1	1
Jul-68	1.918	2.129	1.833	0	-1	1
Aug-68	1.890	2.093	1.578	0	-1	1
Sep-68	1.862	2.058	1.346	0	-1	1
Jan-69	1.985	2.938	5.357	1	-1	1
Feb-69	2.191	3.200	6.615	1	-1	1
Mar-69	2.474	3.430	7.034	1	-1	1
Apr-69	2.567	3.467	7.102	1	-1	1
May-69	2.493	3.242	6.201	1	-1	1
Jun-69	2.432	2.787	4.678	1	-1	1
Jul-69	2.388	2.450	3.566	1	-1	1
Aug-69	2.347	2.323	2.953	1	-1	1
Sep-69	2.304	2.202	2.309	1	-1	1

$$e_1 = \text{if}(\text{and}((Hv_1+Wv_1)>(X_m),(X_m)>(Hv_2+Wv_2)),\text{"1"}, \text{if}(\text{and}((Hv_1+Wv_1)<(X_m),(X_m)<(Hv^2+Wv_2)),\text{"-1"},\text{"0"})) \tag{10}$$

$$e_2 = \text{if}(\text{and}((Hv_3+Wv_3)>(X_l),(X_l)>(Hv_2+Wv_2)),\text{"1"},\text{if}(\text{and}((Hv_3+Wv_3)<(X_l),(X_l)<(Hv^2+Wv_2)),\text{"-1"},\text{"0"})) \tag{11}$$

$$e_3 = \text{if}((Hv_1+Wv_1)>(Hv_3+Wv_3),\text{"1"},\text{"0"}) \tag{12}$$

These equations return either “1”, “-1”, or “0” which Boolean values define presence and direction of flow in each of the edges of the underlying graph.

4. Results and Analysis

The results of each edge were obtained after feeding the mean monthly water levels of Table 3 into the conditional tests set out in equations 10 to 12. The edge results are equally shown in Table 3 against the input data.

The observed water level data reviewed covered 81 months which represented 6.75 years of water level observations used to determine the pattern of flow along the 3 edges of the underlying graph. Over that period there were 26 months in which there was no flow along edge e_1 and only 1 month in which there was no flow along edge e_2.

The 1 month no flow along edge e_2 was unusual given the month, March 1963, as this is the time of the year when there must have been flow along this edge as was observed from the other 80 months. This occurrence could therefore be attributed to a misreading error of the observed water levels at 1 of the nodes on this edge. Edge e_3 had no month in which it was not flowing. There were therefore 54 months in which all the three edges were flowing during the period reviewed.

The results thus present only two main scenarios of the underlying graph, namely, mixed and directed graphs only. Figure 10 and Figure 12 show mixed graphs with no flow along edge e_1 and e_2 respectively. Figure 11 shows a directed graph signifying flow in all the 3 edges. Under each of the graphs (Figure 10 to Figure 12), there is a table (Table 4 to Table 6 respectively) showing months in which that type of graph obtained out of the 81 months reviewed.

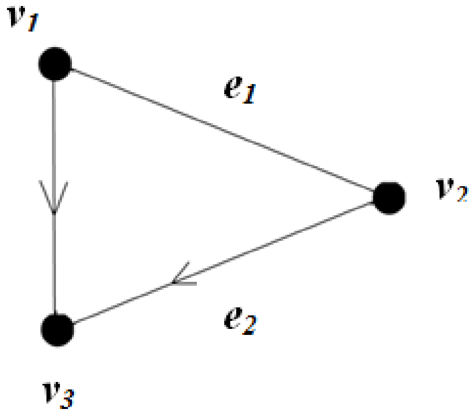

Figure 10. Mixed graph result 1

Table 4. Periods in which Figure 10 graph obtains

	Period	Months	Remarks
1	Oct 62 – Nov 62	2	Data starts in Oct
2	Oct 63	1	
4	Sept 64 – Dec 64	4	
5	July 65 – Dec 65	6	
6	July 66 – Dec 66	6	
7	Aug 67 – Nov 67	4	
8	July 68 – Sept 68	3	Data terminates in Sept
	Total Months	26	

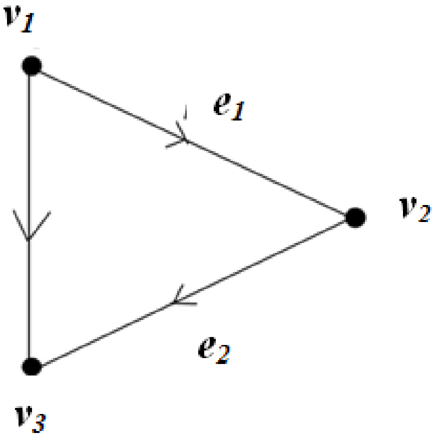

Figure 11. Directed graph result

Table 5. Periods in which Figure 11 graph obtains

	Period	Months	Remarks
1	Dec 62 – Feb 63	3	
2	Apr 63 - Sept 63	6	
4	Nov 63 – Aug 64	10	More flow than usual
5	Jan 65 – June 65	6	
6	Jan 66 – June 66	6	
7	Jan 67 – July 67	7	
8	Dec 67 – June 68	7	
9	Jan 69 – Sept 69	9	Data terminates in Sept
	Total Months	54	

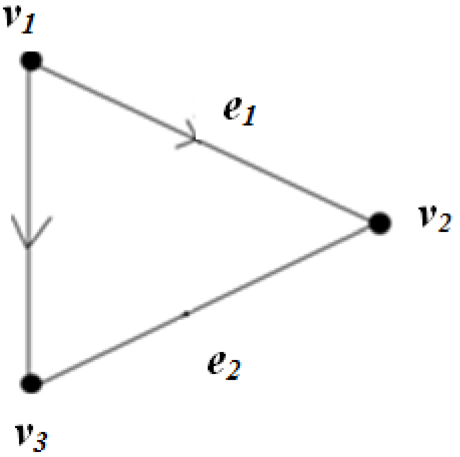

Figure 12. Mixed graph result 2

Table 6. Periods in which Figure 12 graph obtains

	Period	Months	Remarks
1	March 63	1	Probably a misreading error
	Total Months	**1**	

The graph of Figure 12 was most likely a result of an error in the read water levels in 1 of these stations (4425 or 4431) since at this time of the year there should have been flow on edge e_2. This was strongly supported by the fact that this was actually an isolated occurrence out of the 81 scenarios plotted.

5. Discussion and Conclusions

5.1. Discussion

The data used in the surface water exchange model did not indicate any period at which the Lukanga channel, edge e_2, had no flow as such but basically as having a continuous flow along vector v_2v_3 although there was one isolated case when there was no flow which was not collaborated by any supporting evidence. This isolated case was attributed to possible errors in water level data of the two stations involved. The scenarios presented were dependent on water level variability such that it was expected that data from drier years could have shown instances when there was really no flow along this vector or indeed flow in a reverse vector $\boldsymbol{v_2v_3}$ of water along the channel (Figure 7).

The reliability of the model was dependent on both the constants and variables used. As such it was affected by the accuracy of the DEM from which the elevations were derived. In addition the location of the channel lines which were used as profile paths could have been rather inaccurate as user intervention was required where the software could not resolve the path of the channel. This too might have introduced errors in that the profiles were not accurately determined hence their elevations being wrong. These errors could have affected the determined flows.

Water levels were used as variables in the model were from gauging stations which were either at the junctions or considered near enough to the junctions of the network. The overall results of the model could have been affected by use of water level data from gauging stations that were not at the network junctions.

However the overall objective of the study was achieved in that it was demonstrated that presence and direction of flow could be modelled so as to understand the interaction of the surface water from different sources in the area.

5.2. Conclusions

It was deduced from the resulting graphs of the model that (Figure 7):

a) Along edge $\boldsymbol{e_1}$ flow only occurred from vertex $\boldsymbol{v_1}$ to vertex $\boldsymbol{v_2}$ whenever there was flow mostly during the period January to June. There is usually no flow between July and December. It could also be concluded that no flow occurs from $\boldsymbol{v_2}$ to $\boldsymbol{v_1}$ along this edge $\boldsymbol{e_1}$.
b) There is always flow along edge $\boldsymbol{e_2}$ from $\boldsymbol{v_2}$ to $\boldsymbol{v_3}$ throughout the year and nothing in the opposite direction at all.
c) There is always flow along edge $\boldsymbol{e_3}$ from $\boldsymbol{v_1}$ to $\boldsymbol{v_3}$ throughout the year and nothing in the opposite direction at all.
d) All three edges flow at the same time for at least six months with the rest of the time having no flow only along edge $\boldsymbol{e_1}$.
e) Assertions that there was at times bidirectional flow along edges $\boldsymbol{e_1}$ and $\boldsymbol{e_2}$ were found not to be true as is collaborated by the findings of topographical modelling too.

All in all, the water network was identified and established as a graph and modelled from which the water exchange in the network was derived and shown.

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